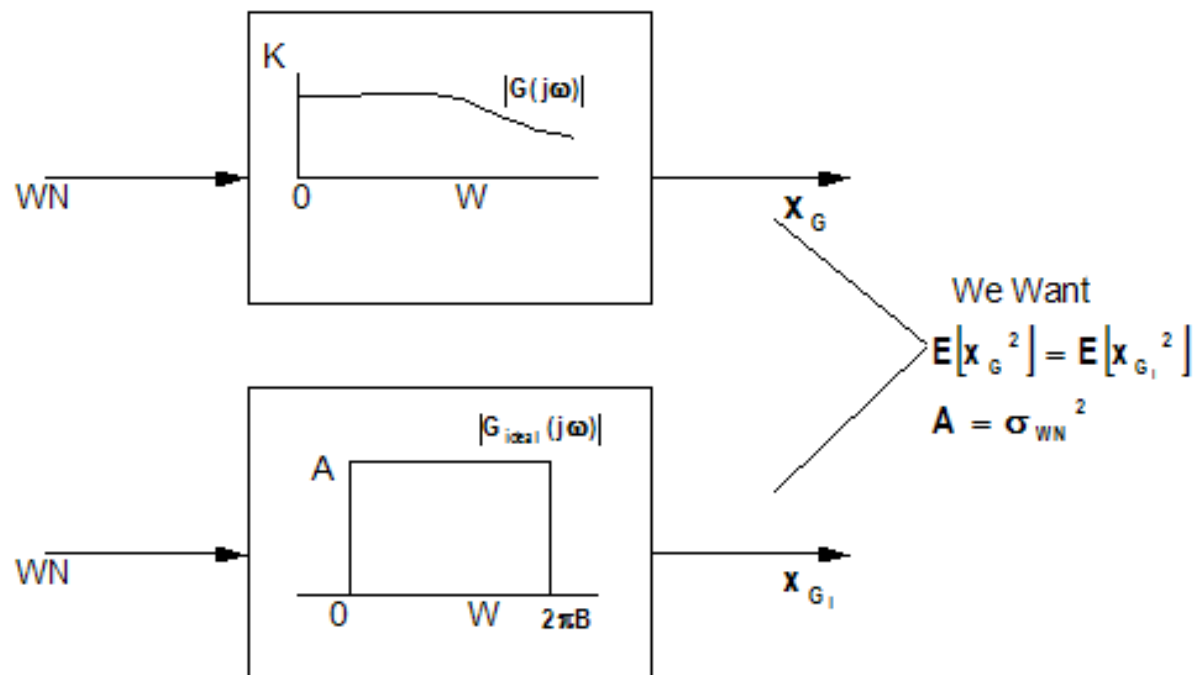


Chapter-3

3.5 Noise Equivalent Bandwidth (Revised 2/26/03)

Definition

$2\pi B$ rad/sec Noise Equivalent Bandwidth is the bandwidth of an ideal filter that yields the same mean square value given that white noise is applied to both filter.



$$E[x_G^2] = \frac{1}{2\pi} \int_{-j\omega}^{j\omega} \sigma_{WN}^2 G(s)G(-s) ds = \frac{\sigma_{WN}^2}{2\pi} \int_{-\infty}^{\infty} |G(j\omega)|^2 d\omega \quad (1)$$

$$E[x_{G_1}^2] = \frac{1}{2\pi} \int_{-2\pi B}^{+2\pi B} \sigma_{WN}^2 d\omega = \sigma_{WN}^2 \omega \Big|_{-2\pi B}^{+2\pi B} = 2\sigma_{WN}^2 B \quad (2)$$

$$2\sigma_{WN}^2 B = \frac{\sigma_{WN}^2}{2\pi} \int_{-\infty}^{\infty} |G(j\omega)|^2 d\omega$$

$$B(\text{Hz}) = \frac{1}{4\pi} \int_{-\infty}^{\infty} |G(j\omega)|^2 d\omega$$

Noise Low Pass Filter (Revised 2/26/03)

If $G(j\omega)$ is a low pass filter then

$$\sigma_{xG}^2 = E[x_G^2] = \frac{K^2 \sigma_{wn}^2}{2\tau} \text{ where } G(s) = \frac{K}{\tau s + 1}$$

$$\text{Hence, } E[x_{G1}^2] = E[x_G^2] \text{ implies that } 2\sigma_{wn}^2 B = \frac{K^2 \sigma_{wn}^2}{2\tau}$$

$$\text{which results in } B(\text{NEB}) = \frac{K^2}{2\tau} \text{ Hz.}$$

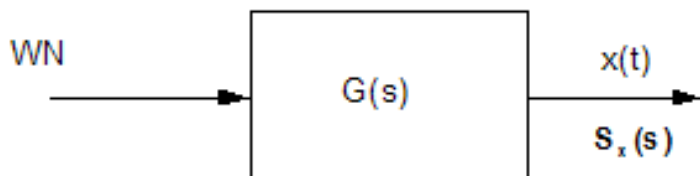
$$\text{IF } K = \sigma_{xG} \sqrt{2\tau}, \text{ then } B(\text{NEB}) = \sigma_{xG}^2 \text{ Hz.}$$

[Click Here To This Page!](#)

Revised 2/9/03

3.6 Continuous-Time Shaping Filter Review

Review



$$S_{WN}(s) = \sigma_{WN}^2$$

The PSD for all real systems can be decomposed as :

$$S_x(s) = \sigma_{WN}^2 G(s)G(-s)$$

We assume $G(s)$ is minimum phase - No zeros of $G(s)$ in right hand plane

Example 3-1, Shaping Filter

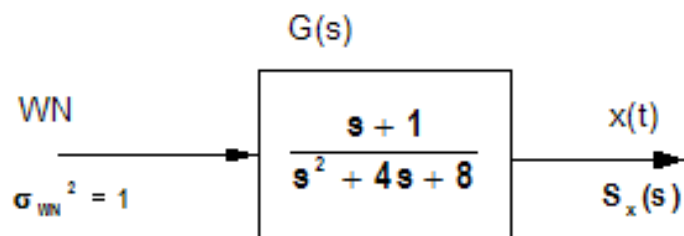
Given a desired continuous-time PSD :

$$S_x(j\omega) = \frac{\omega^2 + 1}{\omega^4 + 64}$$

$$S_x(s) = \frac{-s^2 + 1}{s^4 + 64}$$

Performing spectral factorization on $S_x(s)$ (can always be done for physical systems)

$$S_x(s) = G(s)G(-s) = \left(\frac{s+1}{s^2+4s+8} \right) \left(\frac{-s+1}{s^2-4s+8} \right) \Rightarrow \sigma_{WN}^2 = 1$$



$x(t)$ has the desired PSD characteristics and variance.

3.9 Discrete Process Models

$$\frac{Y(z)}{W(z)} = G(z) = \frac{N(z)}{D(z)}$$

where $N(z)$ and $D(z)$ are polynomials of z

Discrete MA Process / FIR Filter

$$G(z) = \frac{N(z)}{1} \text{ - Moving Average (MA) Process}$$

(Output only a function of input and past input values) & Magnitude of poles and zeroes are less than 1.

Discrete MA Process Example

$$\frac{Y(z)}{W(z)} = 1 + 2z^{-1} + 3z^{-2}$$

$$Y(z) = (1 + 2z^{-1} + 3z^{-2})W(z)$$

$$y(k) = W(k) + 2W(k-1) + 3W(k-2) \text{ - MA Process}$$

An MA Process implies a FIR (Finite Impulse Filter)

Discrete ARMA Process / IIR Filter

$$G(z) = \frac{N(z)}{D(z)}, \quad D(z) \neq 1, \quad \text{Deg}(N(z)) \geq 0 \text{ and } \text{Deg}(N(z)) \leq \text{Deg}(D(z))$$

Autoregressive-Moving Average (ARMA) Process. An ARMA process implies an IIR (Infinite Impulse Filter)

Discrete ARMA Process Example (see Example 3.11 in text)

$$\frac{Y(z)}{W(z)} = \frac{0.5z + 0.25}{z^2 - z + 0.5} = \frac{0.5z^{-1} + 0.25z^{-2}}{1 - z^{-1} + 0.5z^{-2}}$$

$$(1 - z^{-1} + 0.5z^{-2})Y(z) = (0.5z^{-1} + 0.25z^{-2})W(z)$$

$$y(k) - y(k-1) + 0.5y(k-2) = 0.5W(k-1) + 0.25W(k-2)$$

$$y(k) = y(k-1) - 0.5y(k-2) + 0.5W(k-1) + 0.25W(k-2).$$

$y(k)$ is a function of present input, past input, and past output $y(k)$ values.

Chapter-3 Quiz Review

The Chapter-3 Quiz consists of multiple choice or true/false answers. Following are the type of questions and problems on the chapter-3 quiz.

1. Given a Low-Pass Filter and the White-Noise Variance, find its Noise-Equivalent Bandwidth.
2. Given the desired Noise-Equivalent Bandwidth and Low-Pass Filter Output Variance, find $G(s)$.
3. Given a desired Continuous-Time PSD, find the Shaping Filter and White-Noise Variance that will generate $x(t)$ with the desired PSD.
4. Given a Discrete MA process, find $y(k)$ and simulate given initial conditions. Also, estimate $\mathfrak{R}_x(t)$.
5. Given a Discrete ARMA process, find $y(k)$ and simulate given initial conditions. Also, estimate $\mathfrak{R}_x(t)$.

*When you have completed the review above: 1) click on [Chapter-3 Quiz Questions](#) to obtain a list of the problems, 2) solve the problems (you may work together) and 3) then proceed to the WebCT "Homework Quiz" icon, select it and select **Chapter-3 Homework Quiz**. Good Luck!*